Heath.

The High School Algebra Curriculum

THE HIGH SCHOOL ALGEBRA CURRICULUM

BY

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THESIS

FOR THE

DEGREE OF BACHELOR OF ARTS

IN

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Dwight Frederick Heath

ENTITLED The High School Algebra Curriculum

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

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C O N T E N T S

THE HIGH SCHOOL ALGEBRA CURRICULUM

CHAPTER I.

History and Value of Algebra

Algebra as we teach it today is largely the product of the last century, but the very beginnings of the science are to be found nearly five thousand years ago. The papyrus of Ahmes gives evidence of some knowledge of algebra as early as 1700 B. C.[1] Since algebra starts with generalized arithmetic, the latter must necessarily have existed first; but so far as documentary evidence goes, arithmetic and algebra are coeval.[2]

The work in whose title the term algebra appeared, for the first time as far as now known, was written by an Arab.[3] His name was Alkhwarismi, and the title of his work has been noted by different authors in somewhat different ways. One of these is as follows: "Algebr w'almukabala." This work was so well written that it remained a standard for centuries.[4] It is interesting to note that the name of the author, Alkhwarismi, gave rise to our mathematical term algorithm, which is used in general to describe a formal method of procedure to solve mathematical questions. Thus, although Arabic prominence in mathematics lasted only a short time, we are indebted to them for the term algebra which has continued until the present day.

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[1] F. Cajori, History of Elementary Mathematics, p. 19.

[2] Ibid., p. 25.

[3] Miller, p. 83.

[4] Ibid., p. 83.

The Greeks and Romans also had some knowledge of algebraic operations. Diophantus stated a part of our law of signs when he said that "a number to be subtacted multiplied by a number to be subtracted gives a number to be added."[1] But this knowledge was confined to a learned few, and even for them numerical calculations were long and tedious, for the reason that they never possessed the boon of a perfect notation of numbers with its zero and principles of local value.

During the Middle Ages the Hindus made remarkable contributions to algebra. They were the first to recognize the existence of absolutely negative numbers and of irrational numbers.[2] They also took a great step beyond Diophantus in the recognition of two answers for quadratic equations; but although negative roots were seen, they were not admitted, being said to be inadequate.

In the early years of our country algebra seems to have been known very little, if at all. Cajori says, in speaking of Harvard College in 1643, "Algebra was then an unknown science in the New World."[3] Even as late as 1700 algebra was not yet a part of the college curriculum.

It is probable that with the introduction of Ward's Mathematics, algebra began to be studied at Harvard. The second part of the Young Mathematician's Guide (Ward's book)

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[1]F. Cajori, History of Elementary Mathematics, p. 35.

[2]Ibid., p. 101.

[3]F. Cajori, Teaching and History of Mathematics in the U. S., p. 19.

consists of a rudimentary treatise on this subject. It is possible, then, that the teaching of algebra at Cambridge may have begun some time between 1726 and 1738. "But I have found no direct evidence to show that algebra actually was in the college curriculum previous to 1786."[1]

Even when it was introduced into the college curriculum, algebra was taught only in the last year at Harvard. At Yale, in 1742, algebra was recommended to be studied the first year in a regular course of academic studies, followed in the second year by geometry, and in the third year by mathematics.[2] At William and Mary College the subject was taught as early as 1724, but, of course, the treatment was by no means exhaustive, only the rudiments being presented.[3]

All this, however, refers only to algebra in the colleges, and the question arises, When was algebra introduced into the high school? This is a difficult question to answer. In Germany between 1810 and 1830 the gymnasiums assumed the teaching of elementary mathematics, which had before been performed by the universities. Miller,[4] thinks the same transition occurred in our country about the same time. The first mention I find of it is in the catalog of the University of Mississippi for 1859-60, which gives among the requirements

[1] F. Cajori, Teaching and History of Mathematics in the U. S., p. 25.

[2] Ibid., p. 31.

[3] Ibid., p. 34.

[4] Miller, p. 1.

for admission, "algebra as far as simple equations." Since it was required for admission to the University, algebra must have been taught in the secondary schools at least as early as 1860, and probably earlier.

Although the history of algebra in the secondary schools is not immediately available, it is evident that since the time when it was transferred from the college to the high school this subject has become universally a secondary school study. At present every high school in the country offers a course in it at least "to quadratics", and a large majority offer at least "through quadratics". A few even teach college algebra.

For years the study and teaching of algebra were justified on the basis of disciplinary value. But the reactionary tendency has now set in so strongly that one will find few authors today who will try to defend algebra solely or chiefly on the basis of discipline. That there may be some transfer of training from algebra to other subjects cannot be absolutely denied, but the amount possible, as far as experiments have shown, is very small indeed. Bagley says that some transfer may take place if the methods are idealized, and this thought seems to be followed up in the following quotation: "The principle, not the problem, is the heart and soul of mathematics, and a knowledge of principles, together with the ability to think abstractly, that results from right teaching of principles, should be the immediate aim of the algebra teacher."[1] This

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[1]Thought Values in Beginning Algebra, School Review, 1902, pp. 169-184.

suggests that to get the best results stress must be put on the methods as well as on the subject matter. But even with this in mind, we cannot justify the teaching of algebra in the high school merely on disciplinary grounds.

Now that formal discipline no longer forms the sole basis for a justification, many try to justify algebra on the grounds of direct utility. But this also proves to be insufficient. How many pupils ever have occasion in after life to use the knowledge they gained in algebra? For a few, to be sure, who take up technical work in the branches of engineering, it is essential, as well as for the few who follow mathematics, physics, or chemistry as a profession; but what of the vast majority who take up other occupations and professions? Does a knowledge of the binomial theorem help the lawyer to prepare his case, the surgeon to make the proper incision, or the merchant to figure his profits and losses? Is the factor theorem going to help the stenographer, or the law of exponents the clergyman?

From Cincinnati a questionaire was recently sent out to 102 prominent men throughout the country with this question: Which of the following courses in a high school would you advise a boy to take?[1]

1. A course where both mathematics and the classics are optional; for example, where history is substituted for mathematics.

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[1]Educational Research and Statistics, <u>School and Society</u>, June 19, 1915, pp. 893-900.

2. A course where mathematics is required, the classics being optional.

3. A course where the classics are required, mathematics being optional.

4. A course requiring both the classics and mathematics.

The responses were grouped as follows:

Occupation	No. of men	Course 1	2	3	4	2 or 4
Business men	47	3	17	1	21	5
Theologians	19	1	1	-	16	1
Physicians	15	2	1	1	10	1
Lawyers	10	-	3	-	5	2
College heads	3	-	-	-	3	-
Miscellaneous	8	1	3	-	4	-
Totals	102	7	25	2	59	9

The results are overwhelmingly in favor of the courses in which mathematics is required, ninety-three out of 102 favoring either "2" or "4". These answers were not from educators, but from business men and professional men, who, as has already been pointed out, have little or no direct use for the subject matter of algebra. Why then did nine-tenths of them advise a boy to take a course in which mathematics is required?

Hudson stated the ideal motive for studying algebra when he said: "To pursue an intellectual study because it 'pays'

indicates a sordid spirit; of the same nature as that of Simon, who wanted to purchase with money the power of an apostle. The real reason for learning, as it is for teaching algebra, is, that it is a part of Truth, the knowledge of which is its own reward."[1] Few of us, however, can really appreciate this ideal viewpoint. We demand a reason that is more practical. We look for some real value in the subject - some sort of profit that will accrue from its study. Since the direct utility of algebra has been shown to be small, we are forced to turn to culture for the real value of algebra that will justify its place in the high school curriculum.

Why are history, English, and languages taught in the high school? Because they constitute a part of the world's accumulated knowledge that every person should know something about. A knowledge of them is necessary if one is to enjoy to the fullest degree the ordinary happenings of life. Similarly, algebra is also an important branch of truth, and some knowledge of it should be included in the mental equipment of every one who finishes even a part of a high school course.

"A true education should seek to give a knowledge of every branch of truth, slight perhaps, but sound as far as it goes, and sufficient to enable the possessor to sympathize in some degree with those whose privilege it is to acquire a fuller and deeper knowledge. A person who is wholly ignorant of any great subject of knowledge is like one who is born without a

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[1]Hudson, W.H.H., On the Teaching of Elementary Algebra, paper before the Educational Society (London) November 29, 1886.

limb, and is thereby cut off from many of the pleasures and interests of life."[1] Thus one without at least a rudimentary knowledge of algebra is without the means of appreciating the numerous great engineering triumphs, many of the scientific discoveries, and the lives and activities of many of the ancient scientists and philosophers. So, on the basis of culture and general education, the teaching of algebra in the high school can be justified.

To summarize then: Although the doctrine of Formal Discipline has not been entirely overthrown, it no longer serves as adequate grounds for justifying the teaching of algebra. The utilitarian value of algebra to the average person is so small as to be insufficient grounds. Then we must, and can, justify the teaching of algebra in the high school on a cultural basis.

The discussion of the values of algebra is included in this first chapter in order to emphasize the conclusion just drawn, in the light of which the remaining topics, Requirements, Syllabi, and Text Books will be discussed.

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[1] Hudson, W.H.H., On the Teaching of Elementary Algebra, paper before the Educational Society (London) November 29, 1886.

CHAPTER II.

The Choice and Arrangement of Subject Matter

The selection and arrangement of the subject matter for any particular school or class is largely conditioned by the text book used, by the entrance requirements of the colleges and universities, and by published syllabi. These syllabi are published by state departments of education and by associations of teachers who are interested in the subject and who are trying to make the teaching of algebra serve as efficiently as possible the ends already enumerated.

A syllabus is not a hard and fast course of study to be followed by every teacher; it would be difficult indeed to construct such a course as would be suitable for city and rural schools alike in all details, and it would be unjust to the one class or to the other if any one course were required of all alike. All courses in algebra must, however, have much in common whether they be designed for the township, county, or city high school, and the outline of this material together with suggestions as to how to arrange it and present it forms a syllabus in algebra.

The needs and conditions of a high school course differ very markedly in different sections of the country, as one would naturally expect, because of the diversity of geographic, social and commercial conditions in the several localities. Even within the boundaries of the state such a difference may be found to exist. Under these circumstances, unless there is

some effort to standardize the content and to set a minimum content of algebra courses, graduates of high schools in different parts of the state are likely to have quite different, and in some cases very insufficient, algebraic equipment. So, in order to minimize this possibility and to standardize the algebra course for the whole state, several of the states have published syllabi in algebra[1] which give a list of the topics which should be treated and their relative importance.

Other syllabi in algebra have been published by various organizations. Several of them are much more detailed and explanatory than those published by the states and are consequently more useful. A few of these are:

Algebra syllabus adopted by the High School Teachers' Conference of Illinois in 1908. High School Manual, vol. XII, No. 43.

Report of the Committee of the American Mathematical Society on Definitions of College Entrance Requirements in Mathematics. Bulletin of the American Mathematical Society, 1903, pp. 74-77.

The Teachers of Mathematics in the Middle States and Maryland. Syllabus. School Science and Mathematics, December, 1909, p. 200.

Report of the Committee on Algebra in Secondary Schools. Proceedings of the Eighth Meeting of the Central Association of Science and Mathematics Teachers. p. 188.

Syllabus of Mathematics. Society for the Promotion of Engineering Education, 1912, p. 5.

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[1] Such syllabi have been published by Arkansas, Illinois, Indiana, Kansas, Louisiana, Michigan, Missouri, Nebraska, New Jersey, New York, Pennsylvania, and Wisconsin. Most of these syllabi may be obtained by writing to the superintendents of the various states.

American Syllabus in Algebra. School Science and Mathematics, February, 1910, p. 143.

College Entrance Examination Board.

Missouri Society of Teachers of Mathematics.

State Teachers Association of Wisconsin.

Of these the Middle States and Maryland syllabus is by far the most extensive and detailed, and although it deals only with "Elementary and Intermediate Algebra," that is, a one year course, it will make a good standard with which to compare the others. It recommends the consideration of the following topics, no order of presentation being intended:

I. Extension of Arithmetic in Algebra. Positive and Negative Numbers. Definitions. Graphs.

II. Fundamental operations.

III. Factoring.

IV. H. C. F. and L. C. M. by factors.

V. Fractions; reduction, addition, subtraction, multiplication, and division. Complex fractions.

VI. Equations of the first degree in one unknown.

VII. Simultaneous equations in two and three unknowns. Graphs. Problems.

VIII. Involution and Evolution. Square root of polynomials and arithmetical numbers.

IX. Exponents and radicals. Radical equations.

X. Imaginaries.

XI. Quadratic equations in one and several unknowns. Theory. Graphs. Problems.

XII. Binomial theorem for positive integral exponents.

XIII. Inequalities.

XIV. Ratio and Proportion.

XV. Progressions.

This, however, is but the skeleton of the syllabus, and a part of the discussion follows to show in what detail these topics are developed:

I. Extension of Arithmetic in Algebra.

A. Literal numbers as the generalization of arithmetic numbers.

1. Indicated operations, a+b; a-b; axb; a÷b.

2. Powers and fractions result from indicated multiplications and divisions.

3. Minus numbers necessary for a more complete scale of numbers. In arithmetic 3-10 is an impossible operation. It becomes possible as soon as negative numbers are admitted. Introduce the scale ..... -3,-2,-1,0,+1,+2, +3, ... by addition and subtraction. Illustrate by divisions on a line and by as many concrete examples as possible.

4. Simple problems involving the use of literal numbers, e.g., John has <u>a</u> cents and received <u>b</u> cents; how much has he? A letter stands for a number which may be integral, fractional, positive, or negative.

B. Simple Equations.

1. Contrast 30% of cost = $60 with .3x = 60, 5+7 = 12, 19-3 = 22-6; substitute a letter in these examples.

2. Solution of 3x-4 = x+8 by the use of the equality axioms.

3. Discover law of transposition.

4. Literal equations.

a. x+a = b

b. ax+b = c

c. More easy problems.

5. Some very simple problems resulting in numerical simultaneous equations.

Note 1. Extract definitions as they are needed.

Note 2. Graphs of simple forms such as $y = 2x+1$, and $\begin{cases} y = 2x+1 \\ y = 3x+2 \end{cases}$. Use coordinate paper. Measure the $\underline{x}$ and $\underline{y}$ of intersection.

II. Fundamental Operations.

Note. Introduce the laws of signs and exponents and the laws of commutation, association, and distribution as applied to these operations. Some work in detached coefficients should be given.

A. Addition and subtraction.

1. Algebraic addition involves arithmetic addition and subtraction.

2. Meaning of subtraction is to find a number, which, added to the subtrahend, gives the minuend.

3. Removal and introduction of signs of aggregation. Simple cases only. Check by addition. (Illustrative of the laws of addition)

B. Multiplication.

1. Monomials by monomials.

2. Polynomials by monomials.

3. Polynomials by polynomials.

Note. 1. Notion of <u>function</u> and <u>variable</u> may be suggested in this place by the evaluation of a polynomial for values of the letter in it.

Note 2. Laws of homogeneity may be pointed out.

4. Type products.

a. $(x \pm y)^2$, and the square of any polynomial.

b. $(x+y)(x-y)$, $(x+y+z)(x+y-z)$, $(x^2+xy+y^2)(x^2-xy+y^2)$.

c. $(cx^2+a)(cx+b)$.

As has been stated, this is the most detailed syllabus available in published form, and any teacher of algebra may well turn to it for suggestions.

Another very comprehensive syllabus is that of the Society for the Promotion of Engineering Education. It refers especially to a course for prospective engineers and is intended to include "those facts and methods of elementary algebra which a student who has completed a course in that subject should be expected to 'know by heart' - that is, those fundmental principles which he ought to have made so completely a permanent part of his mental equipment that he will never need to 'look them up in a book'". It is not intended as a program of study for beginners, and no attempt has been made to arrange the topics in the order in which they should be taught, but in the hands of a skillful teacher, and supplemented by an adequate collection of problems, it might well be made the basis of a course of study conducted by the "syllabus method".

To give a list of topics to be taught, is, however, not the only purpose of these syllabi. In addition, several of them contain a list of topics, which are included in many text books, which may well be omitted from the first year's work.[1] The following table will show how closely the different syllabi and state recommendations agree in this respect.

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[1]The syllabus of the State Teachers Association of Wisconsin gives twenty-two paragraphs under this heading.

Table II[1]

	a	b	c	d	e	f	g	h	i	j
Cent.Assoc.of Sc. and Math. Teachers	-	-	-	-	-	-	-	-	-	-[2]
Alabama			-						-	-
Arizona	-	-	-	-	-	-	-	-	-	-
Arkansas	-	-	-	-	-	-	-	-	-	-
California			-							
Connecticut							-	-	-	-
Idaho	-	-							-	-
Illinois	-	-	-	-	-	-	-	-	-	-
Indiana	-	-	-	-	-	-	-	-	-	-
Iowa	-	-	-	-	-			-	-	
Kansas	-	-		-		-			-	-
Kentucky	-	-	-			-	-	-	-	-
Louisiana		-						-	-	-
Maine		-	-				-	-	-	-
Maryland		-								
Michigan	-	-	-	-						
Mississippi			-			-	-	-	-	-
Missouri		-	-	-	-	-	-	-	-	-
Nebraska						-				
Nevada			-	-		-	-	-	-	-
New Mexico	-	-	-	-	-	-	-	-	-	-
New York	-	-	-			-				
North Carolina		-	-	-				-	-	-
Ohio		-	-	-				-		
Oregon		-	-	-				-	-	-
Pennsylvania	-	-	-	-	-	-	-	-	-	-
South Dakota	-	-							-	-
Tennessee	-	-	-	-	-	-	-	-	-	-
Utah		-	-	-				-	-	-
Vermont		-		-	-	-	-	-	-	-
Virginia						-	-	-	-	-
Washington	-	-	-	-	-			-	-	-
West Virginia	-	-	-			-		-		-
Wisconsin	-	-	-	-	-	-	-	-	-	-

[1]- indicates that topic is recommended to be omitted.

[2]The Central Association also adds the 'binomial theorem' and 'simultaneous quadratics except one linear and one quadratic!

In Table II, a, b, c, d, e, f, g, h, i, and j denote the following topics to be omitted from the first year's course:

a - Complicated brackets.

b - H. C. D. and L. C. M. by division.

c - Remainder theorem.

d - Complicated complex fractions.

e - Simultaneous equations in more than three unknowns.

f - Cube root of polynomials.

g - Formal study of the theory of exponents.

h - Extended study and manipulation of radicals and imaginaries.

i - Equations containing complicated radicals.

j - Theory of quadratics.

The object of omitting these topics is not to make the course easier but rather to give the course unity and to gain time for the introduction of problems.[1] With these topics omitted, the course affords all the intellectual training that mathematics is supposed to give.[2] The manipulations involved in the above omissions are those that are performed with the least intelligence by first year pupils. Operations performed mechanically give little training in thought and little power, while the solution of problems affords the best possible training.

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[1]Mathematics Teacher, p. 188.

[2]It will be shown in a later section that these topics are omitted from the first course as given in the most recent text books.

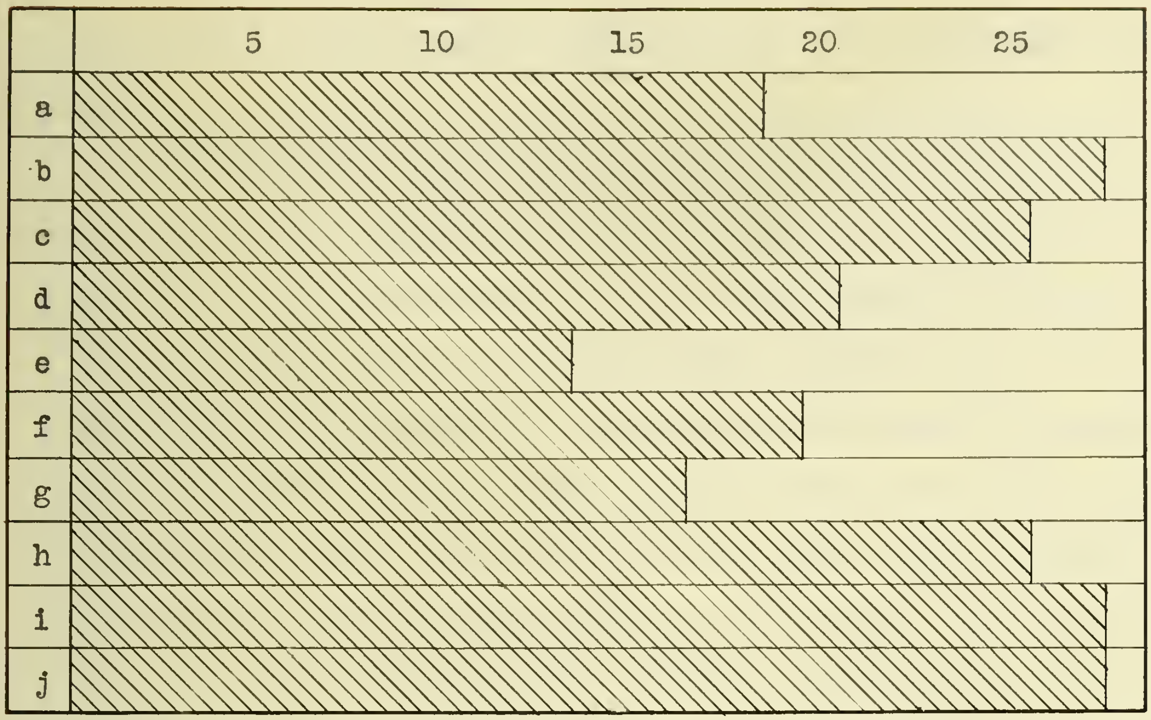

Fig. 1

The data of Table II are summarized in Fig. 1. The shaded areas indicate the number of state recommendations and syllabi that advise the omission of each of the topics listed. The graphical representation is somewhat deceiving at the first glance, because the longer is the shaded area, the less important is the topic considered for first year algebra. It is again misleading because the various syllabi were prepared from nearly as many different viewpoints. Thus (e) "simultaneous equations in more than three unknowns" appears to be the most important, for it has the shortest shaded rectangle to represent it, but the probable reason for this is that the men who prepared the other syllabi took it for granted that the topic would naturally be omitted and so thought it unnecessary to mention it. The same reasoning may hold for some of the other short

rectangles, so one must be very careful in interpreting this figure.

Thus far our consideration has been confined almost entirely to the selection of subject matter. Little has been said about the arrangement of it or the division between the first and later courses as they exist, because the order of presentation of topics will receive rather careful consideration in a later section. The general plan is to give one year of elementary work the first year and then either a year or a half-year of more advanced work in the second, third, or fourth year. In this connection the Illinois syllabus says the first year's course should be "so arranged as to enable the pupil to solve such problems as are within his comprehension and to arouse his interest in algebra as a tool for the solution of problems which are impossible, or very difficult, by unaided arithmetic means," and the second course is "intended to meet the need of those pupils who desire full preparation for college, and comprising a more formal treatment of the principles employed in the first course, together with advanced chapters."

We see then that the syllabus in algebra is a very important aid in the planning of a course in this subject for high schools. As superintendents and teachers come more and more to realize its value and to use it as a guide in making up their courses, the average pupil's working knowledge of algebra is sure to improve. At the same time authors will be forced to fashion their books along the same lines, and the inexperienced teacher will find her work easier.

CHAPTER III.

Requirements in Algebra for High School Graduation and College Entrance

One of the aims of every high school executive is to have his school on the accredited list of first class colleges and universities, so that the graduates of his school may enter these institutions without examination. In order to be thus accredited the school must prepare the student to meet the entrance requirements imposed by the higher institution, and this, in turn, entails the formulation of graduation requirements such that every student registered in a college preparatory course shall, on finishing a four year course, be eligible to college entrance. For this reason the current graduation requirements for our high schools are largely conditioned by the entrance requirements of our universities; so that before considering the present high school requirements it will be well to investigate the college entrance requirements.

In 1912, out of 203 colleges of literature and arts in the United States, nine made no entrance requirement in mathematics, and the others required from two to three and one-half units, 109, or more than half, setting the mark at two and one-half. In this same group of colleges the maximum number of units accepted as entrance credit in mathematics ranged from two to five, with the greatest number of schools accepting four units. Of this group thirty-six were in state

universities, and their requirements range only from two to three units, while the maximum number of units accepted varied from three to five, with three schools making no limit.

In general the state universities west of the Mississippi required only two units. This is perhaps an illustration of the characteristic difference between the East and West. The educational institutions in the East are conservative, and having been established for so long a time, they are still holding to their early customs. As education then was chiefly a matter of culture, a high degree of proficiency in mathematics was thought necessary. But in the West, the product of the last eighty years, there is an air of breadth, freedom, and practicability which is reflected in the college entrance requirements in mathematics.

In the engineering colleges the mathematics entrance requirement was at least a half unit higher, and in the agricultural colleges a half unit lower than in the schools above mentioned. This is naturally to be expected, since the technical courses in engineering need as a basis a broad, well rounded training in mathematics, while the scientific farmer has little, if any, use for mathematics beyond algebra, and there is some doubt as to whether he really needs even that. This information is given below in tabular form which is easily interpreted.[1]

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[1]Data taken from College Entrance Requirements, U. S. Bureau of Education Bulletin, 1913, No. 7 - Whole Number 514.

Table III

Units	203 colleges of L. & A.		36 L. & A. in state univ.		85 colleges of engineering		31 colleges of agriculture	
	req.	acc.	req.	acc.	req.	acc.	req.	acc.
0	9	-	-	-	1	-	2	-
1/2	-	-	-	-	-	-	-	-
1	-	-	-	-	-	-	1	-
1 1/2	-	-	-	-	-	-	-	-
2	53	4	18	-	2	-	11	-
2 1/2	109	15	15	-	9	-	13	-
3	31	38	3	1	51	9	4	2
3 1/2	1	61	-	14	15	27	-	8
4	-	85	-	15	7	49	-	21

More than 4 units counted as 4

High school mathematics is divided as follows:

	units
(a) Elementary algebra	1
(b) Plane geometry	1
(c) Intermediate algebra ...	1/2
(d) Solid geometry	1/2
(e) Trigonometry	1/2
(f) Advanced algebra	1/2

and in these entrance requirements

2 units denote (a) and (b),

2 1/2 " " (a), (b), and (c),

3 " " (a), (b), (c), and (d),

3 1/2 " " (a), (b), (c), (d), (e),

4 " " (a), (b), (c), (d), (e), (f).

An example of the influence of college entrance requirements on the curricula of the high schools is obtainable right here in the state of Illinois, and the incident happened only a year ago. At the University of Illinois in 1913-14 the requirement in algebra was lowered from one and one-half units to one unit.[1] The records are not available to show how many high schools dropped their graduation requirements in algebra the first year in reaction to this change, but probably there were several. During the next year, however, at least three of those schools on the accredited list of the University made this change. These high schools are at Dwight, Carbondale, and Bellflower Township, all of which before 1915 required one and one-half units of algebra for graduation.

This change in entrance requirements has occasioned some little trouble within the University itself. A student may now enter without condition in mathematics with only one year of algebra, while in order to continue in mathematics here he must have had one and one-half years of high school algebra. Now in some courses certain subjects are required for which elementary college mathematics is a prerequisite, and therein lies the difficulty. A student comes to prepare for a professional course in medicine, let us say, and he is required to take a course in physics; but for physics trigo-

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[1]This change from one and one-half to one unit was due to a pressure from the high schools. It was considered unnecessary to burden with extra mathematics those students who were not going to college; those preparing for mathematics, science, or engineering would take the mathematics even if it were not required.

nometry is a prerequisite. The student has entered without conditions and has attended the University for a whole year, and now he meets this difficulty; he cannot take physics unless he registers in trigonometry, and because he has not met the prerequisites prescribed by the Mathematics Department, he cannot take trigonometry. In such a case either the student must make up the deficiency in a local high school, or the Mathematics Department must make an exception in his case; but his condition is also that of several others (and the number will be steadily increasing), and if an exception is made for each there will be little use for the rule. Such a state of affairs is indeed most unfortunate and should be remedied as soon as possible, in order to maintain harmony and coöperation between the high schools and the University.

The only means I can see of clearing up this perplexing situation is to raise the entrance requirement in algebra to one and one-half units. Algebra, although admirably suited to lucid and logical presentation, is but poorly taught in perhaps the majority of our high schools. If students are allowed to enter the University, having had but one year of algebra - and that poorly taught - there will be many in the condition of the medical student mentioned above. He cannot handle readily even simple transpositions in formulas, so how can he be expected to carry successfully advanced work in physics, chemistry, or any other science? Admitting such students will lower the standard of the University, and it is to be hoped that the former requirement will be re-established

very soon.

In Illinois at present the general practice is to offer one year of algebra the first year, and one-half year of advanced work in the third year. Of the 276 high schools accredited by the University of Illinois for which the data were available, 202 make this arrangement, 88 requiring the third half-year, and the other 114 making it elective. Some schools give algebra in the first and second years, completing it before starting geometry, and some defer the advanced course until the fourth year, but all but about fifteen per-cent follow the advice given in the Report of the Committee on Algebra in the Secondary Schools, which says, in referring to the later course, "This course should in no case be given until after demonstrational geometry. The practice, all too common, of completing high school algebra in the first fifteen consecutive months cannot be too strongly deplored. The pupil's introduction to formal proof should be through concrete relations. Moreover, the maturity of mind necessary for this algebraic work is not generally attained before the third year."[1] Evanston Academy and Dixon High School, as well as the technical high schools in Chicago, offer a half year of college algebra in the fourth year. The other extreme is held by three schools which offer only one year of algebra.

These results are put in tabular form below, but it must be remembered that not all the high schools of the state

[1] Proceedings of the Eighth Meeting of the Central Association of Science and Mathematics Teachers, p. 188.

are here represented, but only those that are accredited by the University of Illinois - the better class of high schools. Information about the other high schools, if it were available, would be interesting in so far as it would show whether the tendency among them is the same as that among the accredited ones.

Table IV[1]

No. years offered	No. years required	When given years	No. of schools	Percent total
1 1/2	1	1st, 2nd	9	3.26
1 1/2	1 1/2	1st, 2nd	23	8.33
1 1/2	1	1st, 3rd	114	41.30
1 1/2	1 1/2	1st, 3rd	88	31.88
1 1/2	1	1st, 4th	19	6.88
1 1/2	1 1/2	1st, 4th	13	4.71
1 1/2	1	2nd, 3rd	-	- --
1 1/2	1 1/2	2nd, 3rd	1	- --

Note.--Six schools offer 2 years, 1st and 2nd, 5 requiring 2 years, and 1 requiring 1 year. Three schools offer only 1 year.

Every school considered in this data offers algebra in the first year. And this is as it should be, notwithstanding Professor Judd's theory[2] that algebra should be preceded by demonstrational geometry. "Algebra is a certain science, it proceeds from unimpeachable axioms, it has its own special difficulties but they are not those of weighing in the balance conflicting probable evidence which requires the stronger powers of a mature mind. It is possible for a student to

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[1]The data in this table were compiled from the annual reports for 1915-16 of the accredited high schools of Illinois to the High School Visitor.

[2]Judd: The Psychology of High School Subjects, p. 21.

plant each step firmly before proceeding to the next, nothing is left hazy or in doubt; thus it strengthens the mind and enables it better to master studies of a different nature that are presented later."[1] Thus elementary algebra is well adapted to early presentation, provided the formal proofs of the more difficult theorems are deferred until after demonstrative geometry.

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[1]Smith, p. 171, quoting Dr. Harvey Goodwin in the 19th Century for October, 1886, who quotes Comte, the Positivist Philosopher.

CHAPTER IV.

Recent Text Books in Algebra

In large cities and in well developed school systems the algebra curriculum is very likely to be planned out scientifically and with great care and due consideration of published syllabi and college entrance requirements. In the high schools of our smaller towns, however, and these constitute perhaps the majority of our high schools, little attention is paid to such matters. Here either the teachers are inexperienced or the teaching force is so small that each one has several subjects to teach and consequently has little time for any one of them. In either case the text book is the live-saver to which they cling and which pulls them through in some way. As the ideal text book has not yet been published, the algebra course cannot fail to suffer from such treatment. Then since the text book is responsible to so great an extent for the course in algebra as it is now presented, a discussion of a few recent text books and their points of advantage will be of value in this connection.

Pick up whatever twenty-five year old text book in algebra you please and you will find a development of addition followed by a long list of abstract exercises, then multiplication, subtraction, and division, each with its list of abstract exercises, and in the work for the first half year you encounter nothing of vital importance to the pupil or to his existence or to his activities. But physics, Latin, English, history - near-

ly all subjects are taught differently now from the way they were presented twenty-five years ago. They are connected more closely with the life of today - are more vitally interesting. And so with algebra; writers of text books are coming to realize that the child is interested in events rather than in their explanation and are modeling their books accordingly.

Insertion of illustrations, notes, pictures, and diagrams may be used to add interest, without detracting at all from the value of the book as a text. For instance, when the simple operations with numbers are discussed, a well written note telling how late in the development of the science these simple operations first came to be really understood, and of Sir William Rowan Hamilton's work in this connection[1], adds a great deal of interest to the work, especially if the man's picture is included to give the note a personal touch. When cartesian coordinates are introduced in the discussion of graphs, a short biographical note on René Descartes,[2] who introduced them, proves interesting. The notes need not be all biographical, but any note which adds some touch of interest that would not otherwise be contained in the text or which adds a further motive for the study of any particular division of the subject, will be appreciated by every up-to-date, wide-awake teacher who is really interested in algebra and its future as a high school subject.

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[1]Hawkes, Luby, Touton: First Course in Algebra, pp. 59-60.

[2]Ibid., pp. 199-200.

But the mere insertion of notes and illustrations will not make a successful book. There is another, a much more important factor to be looked for - the arrangement of the subject matter. The subject matter of almost any book can be made at least somewhat interesting and easier to present if it is well arranged, but if the arrangement of the older books is followed, many pupils are likely to lose interest and to look upon the subject merely as a source of work, worry, and trouble. One important remedy for this situation is the early introduction of the simple equation and problems involving it. Instead of drilling on fundamental operations for two or three months before getting into equations and problems, why not introduce the simple equation immediately after the drill on operations on positive numbers? Examples can be so chosen that only positive numbers will be involved, and the pupil will be led to see the advantages of algebraic methods before he encounters negative numbers at all. By thus becoming familiar with the equation early, the pupil is prepared to solve problems, and this adds interest and motive to the further work.

Another point of improvement in recent books over the older ones is the introduction of negative numbers by means of a scale. Subtraction and multiplication of negative numbers have always been a stumbling block to beginners in algebra, and any device that will help to obviate this difficulty should be well considered. Negative numbers appear <u>absurd</u> or <u>fictitious</u> so long as there is no visual or graphical

respresentation of them. By proper use of a scale of positive and negative numbers it can be made clear to even the slowest pupil that subtraction of a negative number is equivalent to addition of a positive number of the same absolute value. The scale should of course be augmented by illustrations and applications to temperature above and below zero, north and south latitude, east and west longitude, and profit and loss; but the scale is most important for the understanding of negative numbers.

The graph may be used to add further interest to the first year's work. A picture of a problem or principle always makes it easier to remember. The graph of such an equation as $y = x+c$ will make clear to the beginner why we call $y = x+c$ a linear equation. In the solution of simultaneous linear equations, also, the graph makes the solution something real to the pupil. Graphical representations of statistics are to be encountered constantly in the newspapers, and magazines, and in books, and a rather intimate knowledge of their use and interpretation are necessary for an intelligent understanling of every day reading material. So for this reason also, the pupil should be given a good working knowledge of graphs, and that as early as possible. Of seventeen elementary text books considered in a tabular statement later on, all but Milne's Academic Algebra give some consideration to graphs but in four of these the chapter is just 'stuck in' without any regard to the development of the other topics, and a few more use graphs only incidentally elsewhere. Stone-Millis's First Course is the

best example of the extensive use of graphs and illustrations throughout the book in the development of other topics, but I believe even this can be improved and the graph made even more an inseparable part of elementary algebra.

So closely connected with the graph that we can hardly think of the latter without it is the function notion. To be sure, there are graphical representations, such as appear in this chapter for example, in which no functional relationship is shown, because the elements along the base line are arranged in arbitrary order and are entirely independent of each other. But in elementary algebra nearly all our graphs are simply pictures of a functional relationship between the variable plotted as ordinates and that plotted as abscissas. Consequently the function should be considered even before the graph.

The function is one of the most common and useful concepts of life - life itself is a complex function of many variables. Our conduct is a function of variable circumstances; the blooming of the flowers is a function of the variable weather conditions. The function is the kind of thing the pupil will want to think about a great deal of his life. If he thinks about it clearly and accurately during his study of algebra, it cannot fail to be of use to him later in life. The function concept furnishes a unifying principle on which to build much of the elementary and advanced mathematics, and helps to lend both concreteness and coherence to the subject.

Another difference between the new and the old books

is found in the method of simplifying complex fractions. Formerly a complex fraction was considered merely as an expressed division of one fraction by another and treated as such, the method of simplification being to multiply the numerator by the inverted denominator.[1] This method is all right for some cases where the numerator and denominator are both simple fractions; but in a case where either is composed of two or more fractions it becomes necessary first to reduce them to a common denominator and then to perform the multiplication. Now this involves two operations when the same purpose might just as well be served by one. By multiplying all terms in both numerator and denominator by a quantity that will make all the minor denominators disappear, any complex fractions can be simplified in one operation.[2] But although the latter method is preferable and the former workable, it is hardly advisable to teach both methods to first year pupils, lest they master neither. It is better to teach one method thoroughly the first year and leave the other until the later course.

While we are discussing the advantages of the newer books, their treatment of the extension of the number system should be given a little attention. Most of the books, both new and old, treat negative numbers early; then later they

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[1]This method is called the 'Old Method' in a later discussion.

[2]This method is called the 'New Method' in a later discussion.

give some space to fractions, irrationals, and imaginary numbers, all in separate chapters. Thus while each of these divisions may be well enough discussed, the pupil gets no idea of the number system of algebra as a whole or the reasons for its extension. Imaginaries were unfortunately named, but that should be only further reason for making them clearer to the pupil. To say simply that an imaginary is an even root of a negative number makes it very unreal to the beginner, and such treatment makes further handling of them dull drill without reason. A separate chapter devoted entirely to the extension of the number system should show that negative numbers were added to the positive numbers of arithmetic in order that subtraction might hold for all numbers, and that fractions were invented so that division might be universal. It should explain also that irrationals and imaginaries serve the purpose of making our number system symmetrical to the extent that every number has a square root, and every quadratic equation two roots. Slaught and Lennes's first book and Rietz, Crathorne and Taylor's second book thus devote a special chapter to the subject.

Another very important consideration is that of the problems and exercises. It is easy enough to develop a given topic in class and then assign twenty abstract exercises as a drill lesson for home-work; but such a procedure is a waste of time and does not consider the interest of the pupils. The recent books are avoiding this error by giving enough abstract exercises to fix firmly in mind the operation in question but not so many as to make the drill tiresome or superfluous. To

fill in the gap left by the omitted exercises, word problems are substituted. By word problems I mean those in which the conditions are written out and the pupil must formulate his own symbolic statement as well as plan his method of attack. Such problems give the same sort of drill as the abstract exercises, at the same time causing the pupil to think and to use his analytical reasoning powers,and therein lies their value.

But the kind, as well as the number, of such problems is important. The average first year high school pupil is a lively, growing boy or girl who is interested in doing things. 'Made up' problems that are utterly impossible do not interest such a pupil as do 'actual' problems derived from fields in which he is vitally interested. The recent books contain problems from physics, chemistry, banking, baseball, basketball, track events, swimming, and other such activities in which the pupils are or will be especially interested.

In the presentation of these problems by the pupils, reasons and methods should be emphasized rather than results. The average teacher regards ability to solve problems as the end in algebra work and drill as the means to this end.[1] Consequently the pupils think it means only drill and grind and they work for the answer. A few problems with emphasis on the "why" and "how" of each step excite much more thought and reasoning than a large number "worked for the answers". Smith emphasizes the exercise in logic. He says, "To be able to

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[1]Thought Values in Beginning Algebra, School Review, 1902, pp. 169-184.

extract the fourth root of $x^4+4x^3+6x^2+4x+1$ is a matter of very little moment. The pupil cannot use the result, nor will he be liable to use the process in his subsequent work in algebra. But that he should have the power to grasp the logic involved in extracting this root is very important, for it is this very mental power, with its attendant habit of concentration, with its antagonism to wool-gathering, that we should seek to foster."[1]

We learn things thoroughly, not all at once, but by recurring to them time after time. By the end of the term or of the year the first year pupil will have forgotten some of the operations that were supposed to be thoroughly familiar to him, unless sufficient opportunity is given him to refresh his memory concerning them. This end may be attained easily and successfully by frequent cumulative reviews, which should occur preferably at the end of each chapter. The problems in these reviews should involve all the principles and rules that have been treated up to that point and their application to situations other than those previously encountered. So often the pupil associates a given principle with a certain set of exercises or with some page in the book, and so long as he knows a problem depends upon that principle he can turn to the page and see what method to use; but if he is not told to what group it belongs, he is powerless to attack it at all. So a cumulative review, if well arranged, should not only recall the preceding principles and rules, but should also call for their

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[1] Smith: <u>The Teaching of Elementary Mathematics</u>, p. 168.

application in a different setting than that previously encountered.

When the pupil is working problems he should not ordinarily have the answer before him. It is too great a temptation to "work backwards". I have noticed physics books with the answers to the problems right after the respective problems, but I do not remember having ever seen a beginning algebra book so arranged. But although the answer should not appear with the problem, in many cases it is well to have it given somewhere so that the pupil may check the accuracy of his work. Of seventeen books investigated, two give all the answers, two some, and several have them in separate answer books.[1]

Aside from the subject matter there are some mechanical points of algebra text books to be considered. Among these are the indexing and labelling of definitions, rules, and principles. If a book is to be of any value as a reference book the index must be good so that the pupil can tell just where to turn for his subject matter. When he turns to the right page he should be able to pick out immediately what he wants, and he can if things are well labelled. In three books I find no index at all![2]

The points herein mentioned and a few others for seventeen recent text books have been put in tabular form (Table V). The numbers across the top represent the books:

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[1]Wells and Hart's new two volume and one volume books not considered above can be had either with or without answers or with separate answer book.

[2]See next page.

1. Slaught and Lennes: First Principles of Algebra.- Elementary Course.

2. Slaught and Lennes: High School Algebra.- Advanced Course.

3. Young and Jackson: Elementary Algebra.

4. Rietz, Crathorne and Taylor: School Algebra.- First Course.

5. Rietz, Crathorne, and Taylor: School Algebra.- Second Course.

6. Kent: A First Course in Algebra.

7. Cajori and Odell: Elementary Algebra.- First Year Course.

8. Somerville: Elementary Algebra.

9. Stone-Millis: Essentials of Algebra.- First Course.

10. Stone-Millis: Essentials of Algebra.- Second Course.

11. Wells and Hart: New High School Algebra.

12. Collins: Advanced Algebra.

13. Wentworth: Elementary Algebra.

14. Milne: Academic Algebra.

15. Hawkes-Luby-Touton: First Course in Algebra.

16. Hawkes-Luby-Touton: Second Course in Algebra.

17. Wentworth-Smith: Academic Algebra.

The letters in the left hand column represent these questions:

a. Is the extension of the number system considered in a special chapter?

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[2]Slaught and Lennes: Advanced Course.
Young and Jackson: Elementary Algebra.
Milne: Academic Algebra.

b. Is division by zero barred?

c. Is the zero exponent explained?

d. Are negative numbers introduced by means of a scale?

e. Is the <u>need</u> of imaginary numbers shown?

f. Is the distinction between equation and identity clearly pointed out?

g. Are complex fractions cleared by the old (O) or the new (N) method or both (B)?

h. Is the monomial factor sufficiently emphasized for later use in collecting coefficients?

i. Are graphs treated?

j. Are they used throughout the book in developing other topics?

k. Are the laws of exponents stated in words as well as in symbols?

l. Are they thus stated in a group?

m. Are logarithms taken up?

n. Are determinants taken up?

o. Are some (S) or all (A) answers given?

p. Word problems - many (M), sufficient (S), few (F)?

q. Abstract exercises - many (M), sufficient (S), few (F)?

r. Is there a cumulative review?

s. Historical notes? Many (M), few (F)?

t. Are principles labelled and easy to find?

u. Is the 'how' and 'why' distinction made between rule and principle?

v. Are definitions labelled as such?

w. Are progressions treated?

x. Are series treated?

y. Is the theory of quadratic equations considered?

z. Index?

The results of Table V are shown graphicallly in Fig. 2, in which the length of the shaded sections indicates the number of affirmative answers to the different questions. From this figure it is seen that only two books devote a separate chapter to developing the extension of the number system. Division by zero is definitely ruled out by only eleven, though some of the others may imply it in their check of division of polynomials. Ten have adopted the linear scale as the best means of introducing negative numbers. Fifteen give a good explanation of the zero exponent. But only five adequately show the need of imaginary numbers. Thirteen distinguish clearly between equation and identity.

The monomial factor is very important when we come to collecting coefficients, but only ten books sufficiently emphasize it for practical use in this connection later on. Although sixteen give some discussion of graphs, only twelve use them in developing other topics, and few use them consistently wherever possible. Sixteen, all but Hawkes-Luby-Touton's Second Course,[1] state the laws of exponents in words as well as in symbols, but only Collins' Advanced Algebra takes advantage of the opportunity for fixing them in the pupil's mind that is offered by a grouping of the laws thus stated.

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[1]They are thus stated in their First Course.

Table V[1]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
a	-	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	X
b	-	-	-	-	-	-	X	X	X	X	-	-	X	X	-	-	-
c	-	X	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-
d	-	X	X	-	X	-	-	-	X	X	-	X	-	-	-	X	-
e	X	X	X	X	-	X	-	-	X	-	-	X	X	X	X	X	X
f	-	X	-	-	-	-	X	-	-	-	-	X	-	X	-	-	-
g	B	B	B	N	B	B	N	B	B	O	O	B	B	B	O	O	B
h	X	X	X	-	X	X	X	-	-	-	X	-	-	-	-	-	-
i	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-
j	X	X	-	-	-	-	-	X	-	-	-	-	-	X	-	-	X
k	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-
l	X	X	X	X	X	X	X	X	X	X	X	-	X	X	X	X	X
m	X	-	-	X	-	X	X	-	X	-	-	-	-	-	X	-	-
n	X	X	X	X	-	X	X	X	X	-	-	-	X	-	X	-	X
o	X	X	X	X	X	S	X	X	A	A	S	X	X	X	X	X	X
p	S	S	F	S	F	M	S	M	S	S	S	S	S	F	S	S	F
q	M	S	M	M	S	F	S	S	M	S	M	F	M	M	M	F	M
r	-	X	-	-	-	-	X	-	-	-	X	X	-	-	X	X	-
s	F	X	X	F	F	X	M[2]	X	X	X	F	M[3]	X	X	M	M	F[4]
t	-	X	X	X	X	-	X	X	X	X	-	-	X	-	-	-	X
u	X	X	X	X	X	-	X	X	X	X	X[5]	-[6]	X	-	-	-	X
v	-	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
w	X	-	-	X	-	X	X	-	X	-	-	-	-	-	X	-	-
x	X	X	-	X	X	X	X	-	X	-	X	-	-	-	X	-	-
y	X	X	X	X	X	X	X	-	X	-	X	-	X	-	X	-	X
z	-	X	X	-	-	-	-	-	-	-	-	-	-	X	-	-	-

For notes on Table V, see p. 41.

Logarithms are discussed in only eleven, but this is because the other six are either first volumes of two volume series or other books intended only for the first year's work, from which logarithms should be omitted in favor of further drill and application of other more elementary topics. Determinants, also, are treated only in those books that include the advanced work, and in but six of them. As has been already mentioned, only four books give any answers at all, and two of these do not give all of them. In the case of exercises and problems, a majority of the books contain what I have termed "sufficient" abstract exercises for drill purposes and "many" word and thought problems. Eleven give some sort of review either at the end of every chapter, at the end of every three or four chapters, or at the very end of the book.

Eight of the books contain instructive and interesting historical and biographical notes, while Wentworth-Smith's Academic Algebra makes up for them by inserting at the back of the volume a four page History of Algebra. Seven label principles Principle, but only two mark definitions Definition, and although most of the books are fairly well indexed, three

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[1]'-' indicates affirmative, 'X' negative.

[2]Cajori is given as authority for notes.

[3]Ball and Cajori are given as authorities.

[4]A four page History of Algebra is given at the back of the book.

[5]All are called Rule.

[6]Principles are called theorems, rules are called laws.

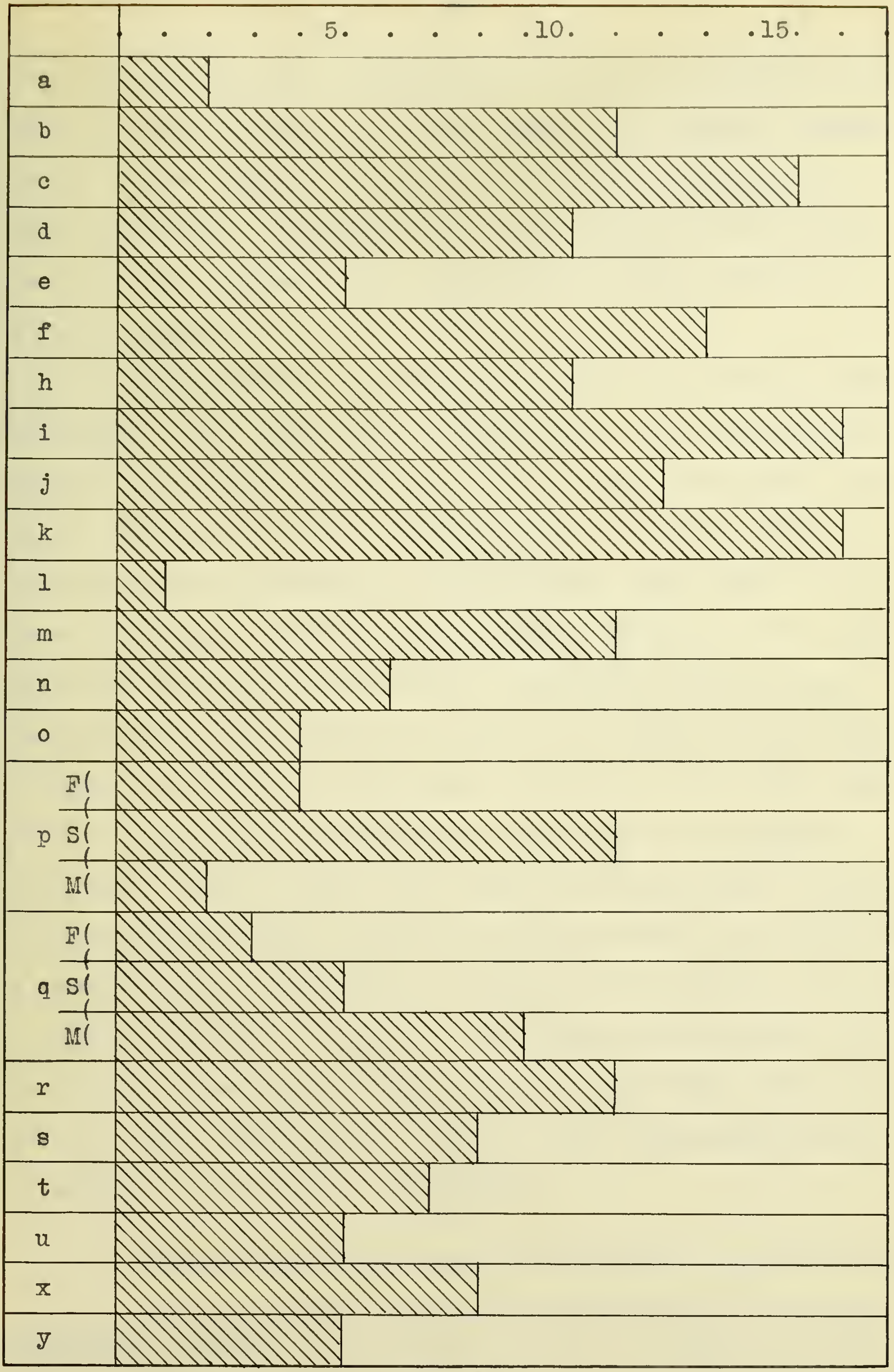

Fig. 2

have no index at all. A very good distinction between rules and principles is that the former answer the question "How?", and the latter "Why?", but only five of the seventeen make this distinction. Some call all rules, some all principles, some laws and theorems, but most make no such distinction as is here suggested.

The perfect text book in algebra has not yet been published. Text books must be marketable, and conservative teachers and school boards are slow to adopt books with very radical changes in them;[1] consequently the changes must be made slowly and gradually. But I believe that within fifteen years more Fig. 2 will be shaded almost solidly to the right hand edge, and other more advanced questions will be under consideration. But since the perfect book has not yet been published, we must get along with the best we can find, and each of us may look for different qualities in an algebra text book.[2] Fig. 3 shows what books are being used this year in 242 accredited high schools in Illinois and how widely each is used.[3] Rietz, Crathorne and Taylor's two volumes are found in only one school of this list this year, because they were not off the press in time for others to get them, but in the next two years their place in Fig. 3 will undoubtedly drop to nearly, if not quite, the bottom.

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[1]State laws also forbid too frequent changes of texts.

[2]A list of 126 text books for algebra is given in Mathematics Teacher for September, 1915, v. VIII, pp. 32-35.

[3]This information was obtained from the annual reports for 1915-16 of the accredited high schools of Illinois.

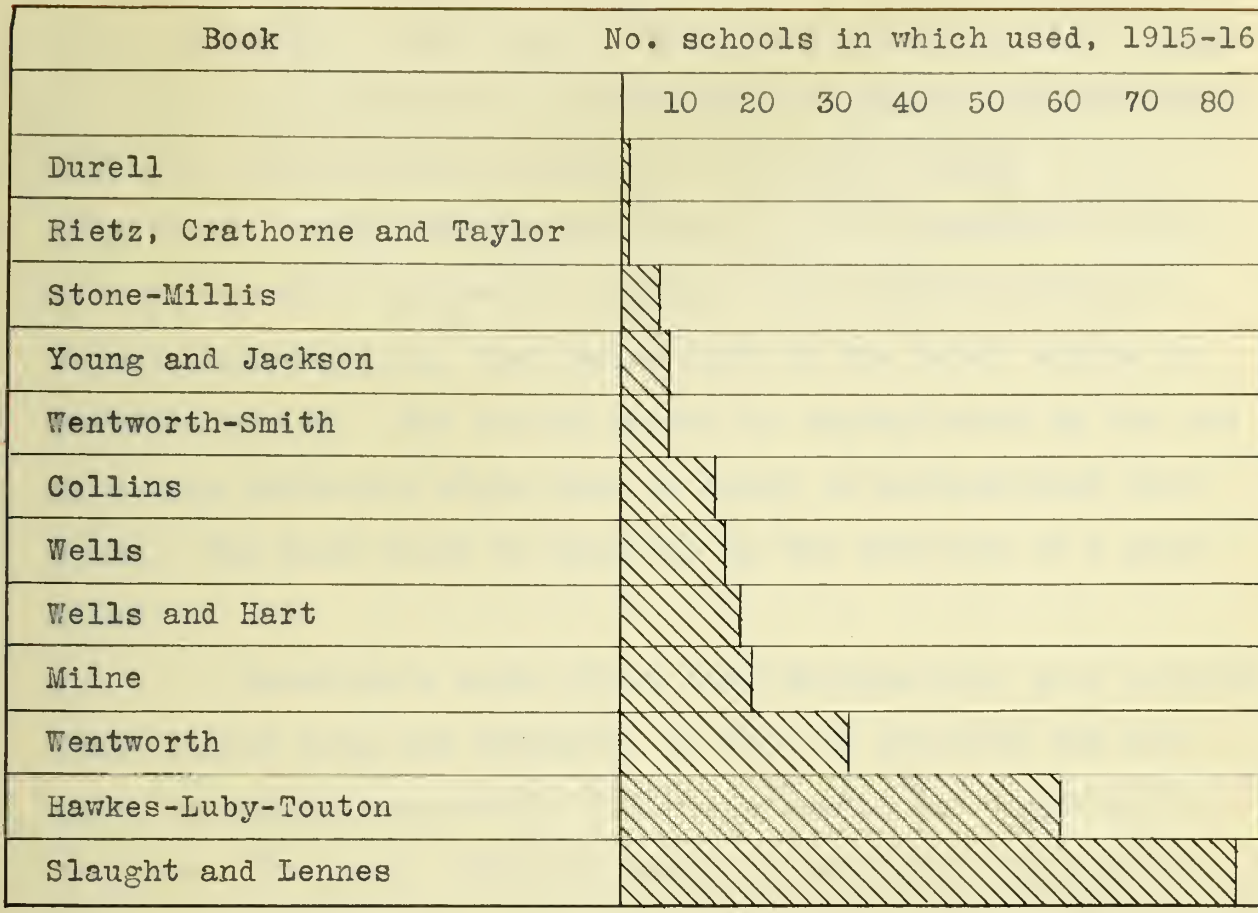

Fig. 3

In closing let us consider briefly a new departure in high school mathematics - correlated mathematics. Probably the two most prominent exponents of this movement in America are Miss Edith Long of Lincoln, Nebraska, and Breslich of University High School, Chicago. The former with Professor W. C. Brenke of the University of Nebraska published in 1913 a First Course in Algebra which is somewhat different from the other text books that we have considered. In the preparation of the book the authors claim to have had two primary aims, "first to emphasize and vivify the treatment of Algebra by a systematic correlation with Geometry, and secondly to present the subject-matter in a style sufficiently simple to be easily grasped by

students of high school age." The first object is accomplished by a free introduction of constructive exercises from geometry, including theorems and problems of sufficient range to give to the student a fair working knowledge of the elementary properties of important geometric figures. No attempt is made at formal demonstration, that being left to the later course in geometry proper. The second object is accomplished by the use of a more narrative style than is usual in mathematical text books. The book would be improved by the addition of a good index.

Breslich's book, First Year Mathematics, goes several steps beyond Long and Brenke's, in that it presents the elements of algebra, geometry, and trigonometry all together in one year. The more difficult parts of each are, of course, postponed until the second volume (now in press) and later differentiated courses. This method of presentation emphasizes the relation between and the inter-dependence of the three usually distinct branches of secondary mathematics. The pupil is introduced to all three and becomes interested in further study of them, whereas under the prevailing system he learns only algebra the first year and often becomes discouraged so that he drops any idea of further study. This book has passed the experimental stage. The method has been developing under the guiding hand of Mr. Breslich at the University High School since 1903, and this fourth edition, published in September, 1915, bears evidence to its success.

Aside from the method of presentation, this book has

many commendable qualities. At the very beginning the pupil is given a page and a half of suggestions to help him in his study of the subject; this is entirely new for mathematics text books. The text is rich in portraits and biographical notes that are very interestingly written. The book is written in a simple, direct style, and finally, it is very well indexed.

Both of these books seem very radical to most of us at first, but they certainly contain much food for thought and consideration. It would not be surprising if they were but mile-stones along the path of advance in the teaching of secondary mathematics, and probably as we "grow to them" they will seem perfectly natural to us.

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